Submesoscale dynamics in tropical instability waves

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Submesoscale dynamics in tropical instability waves (TIWs) of the Pacific ocean are analyzed from nested numerical simulations of increasing resolution (36 km, 12 km, and 4 km). It is shown that numerical convergence, as determined by an invariance of the kinetic energy (KE) spectrum with resolution, can be obtained for grid spacing around 10 km. This finding contrasts with mid-latitude simulations of submesoscale processes that do not converge even for grid spacing less than 1 km. The difference is associated with the larger Rossby radius of deformation at low latitudes due to the weaker Coriolis parameter. For the same reason, the mixed layer Rossby radius is larger as well, resulting in submesoscale mixed layer eddies (MLEs) with scales of 50–200 km. Similar to MLEs at mid-latitudes, those eddies that form at TIW fronts re-stratify the mixed layer by releasing available potential energy. They lead to an additional source of KE at submesoscale that is larger in the finer resolution simulations. At wavelengths smaller than the mesoscale peak of KE injection, a forward cascade of KE is evident in the simulations where three dynamical ranges are observed: a quasi-inertial range of slope \( k^{-2} \), a pre-dissipation range and a far-dissipation range. Numerical dissipation is evaluated and significant dissipation is found to occur in a pre-dissipation range, i.e., for wavenumbers well below the range where the KE spectrum tails off exponentially.

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1. Introduction

In recent years, understanding of oceanic submesoscale dynamics has significantly improved (see Thomas et al., 2008, for a review). While still investigating its specific role, in particular in biogeochemical transport, we now acknowledge the ubiquity of upper ocean frontal dynamics in many mid- and high-latitude oceanic regions. Because typical scales across surface fronts are well below the deformation radius, this dynamics is called submesoscale. A potentially important aspect of submesoscale dynamics is its role in the energetic functioning of the upper ocean. It often responds vigorously to atmospheric forcing and there is a strong indication that it mediates to some extent the process and quantity of buoyancy, momentum and energy penetration into the deeper ocean. Going beyond our present knowledge requires a better description of submesoscale regimes and understanding of the connection between submeso- and microscales (weakly influenced by earth rotation). This study contributes to both aspects by (i) documenting the type of submesoscale that populates an equatorial region (ii) addressing the issue of frontal statistical arrest (Capet et al., 2008a,c; Taylor and Ferrari, 2009), and particularly, the convergence of kinetic energy injection associated with frontal intensification and baroclinic instability. The latter is a salient issue because vertical velocities and tracer fluxes are not directly measurable so that their estimation, using prognostic or diagnostic numerical methods, is sensitive to model/data resolution.

The conceptual framework for this study is largely inspired by the works of Capet et al. (2008c), Klein et al. (2008), and Molemaker et al. (2010). The increase of numerical resolution toward the kilometer scale (or improvement of observational data coverage and treatment; Le Traon et al., 2008) invariably leads to flatter surface KE spectra than those expected from quasi-geostrophic mesoscale dynamics: spectral slope is closer to \(-2\) than \(-3\) or steeper. KE injection initiated by mesoscale straining and possibly reinforced by submesoscale instability is the underlying mechanism that produces shallower spectra. Although intensified at the submesoscale, this injection occurs over a broad range of scales where the existence of a true inertial cascade is therefore precluded. Advection nonetheless plays an important role by fluxing KE energy upward from mesoscale to large scale (inverse cascade) and downward to submesoscale (forward cascade). Consequently, part of the kinetic energy released from available potential energy (APE) leaks toward smaller scales, en route to dissipation (Molemaker et al., 2010); in the process, it participates in surface restratification. Although this leakage is presumably important in the upper
ocean only, remaining uncertainties on the oceanic energy cycle provide a strong motivation for further investigation.

This conceptual framework is consistent with other modeling studies and also with few observational evidences, all of them for mid-latitude regions or idealized situations. To our knowledge, the only energy budget for a non-quasi-geostrophic numerical solution having converged is presented in Molemaker et al. (2010). Dimensionalized for mid-latitudes, frontal arrest occurs in these simulations at a horizontal resolution of about 250 m. Mixed layer instabilities (MLI) that are known to invigorate submesoscale turbulence (Boccaletti et al., 2007) are not permitted in the idealized configuration of Molemaker et al. (2010), but theoretical arguments suggest that their results would hold true using more realistic models. However, since the Coriolis frequency enters the expression for the mesoscale and submesoscale basic eddy lengths (baroclinic and mixed layer deformation radii), we anticipate a shift toward larger values for all other length scales in the problem, in particular the inverse/forward flux separation scale and the resolution required to reach frontal arrest (in a statistical sense). From altimeter data, such a latitude effect on the spectral energy density and flux has been evidenced at mesoscale by Scott and Wang (2005), but no investigation of this effect on the submesoscale spectral energy balance has yet been documented.

The near-equatorial band combines large changes of the Coriolis frequency with intense mesoscale straining due to tropical instabilities. Tropical instability waves (TIWs) are common to the Pacific and Atlantic ocean, routinely observed in space-borne scatterometer data, provided by CERSAT (www.ifremer.fr/cersat) on a 1° grid for the period 1993–2000. At the three simulations of variable resolution are proposed:

- **FINE**: 4 km resolution grid encompassing one selected TIW, embedded into 3 coarser grids.
- **MEDIUM**: 12 km resolution grid encompassing three TIWs, embedded into 2 coarser grids.
- **COARSE**: 36 km resolution grid encompassing the main TIW formation area, embedded into the 1° resolution grid.

The resolution sensitivity (at 4 km, 12 km and 36 km resolutions) serves as a measure of the convergence level of the model solution. The 1° resolution grid is not used for comparison as it is too coarse for proper spectral diagnostics to be made (a few grid points per wave). The model grids, forcing, initial and boundary conditions are built using the ROMSTOOLS package (Penven et al., 2006). The bottom topography is derived from a 2° resolution database from NOAA-NGDC (ETOPO2). Although pressure gradient errors in ROMS are handled by a high-order scheme (Shchepetkin and McWilliams, 2003), bathymetry smoothing that maintains a upper limit of 0.2 for the slope parameter Δh/2h is still needed. In any case, the bathymetry in the eastern equatorial Pacific is naturally smooth and flat and does not play any significant role in our analysis. To preserve a sufficient resolution in the upper ocean, we use 30 vertical levels with surface-stretched s-coordinates (parameters \(s_1 = 6, s_2 = 0\)). The surface layer resolution is about 5 m and a maximum grid size of about 500 m is in the bottom layer.

The model external forcing is derived from global model reanalysis, simulations, and satellite observations. At the surface, the model heat and fresh water fluxes are extracted from the COADS climatology (da Silva et al., 1994). Momentum fluxes are interpolated from the weekly wind stress product derived from the ERS satellite scatterometer data, provided by CERSAT (www.ifremer.fr/cersat) on a 1° grid for the period 1993–2000. At the three lateral boundaries facing the open ocean, a mixed passive-active radiation condition connects the model solution to the surroundings (Marchesiello et al., 2001). In the case of inflow conditions, the solution at the boundary is nudged toward monthly outputs of the ORCA05 global ocean model, which was run on a 1/12° resolution grid for the period 1992–2002 (Couvelard et al., 2008).
Finally, initial conditions in the FINE, MEDIUM and COARSE simulations are interpolated from the equilibrated 1° model solution after about six years of spin up (1993–1998) starting with ORCA05 January mean values. The analyses are performed using three month long simulations starting from September 1998 (hereafter noted ‘high TIW season’), i.e., during the 1998 La Nina event. During the three month period, three TIWs were able to cross the finest grid domain.

### 3. Results and analyses

#### 3.1. Equatorial dynamics and mesoscale structure

A complete model verification of equatorial dynamics and TIW activity is beyond the scope of this paper which is devoted to the sub-mesoscale. However, submesoscale dynamics partly results from straining by mesoscale eddies, which thus require a minimum validation. 1993–2000 mean characteristics of equatorial dynamics are presented for the COARSE solution in Figs. 1–4, where they are compared with drifter estimations (Lumpkin and Garraffo, 2005) and TOGA-TAO current observations (McPhaden and McCarty, 1992).

The mean equatorial current system is composed of the eastward flowing North Equatorial Undercurrent (NEUC) and Equatorial Undercurrent (EUC) and the westward flowing South and North Equatorial Counter-Currents (SEC and NEC). They are represented in the model with realistic structure and magnitude within uncertainties of wind forcing sensitivity (Cravatte and Menkes, 2009). The realistic model representation of mean horizontal and vertical flow shears is also associated with realistic mean Eddy Kinetic Energy (EKE) along the equatorial guide compared to drifter observations (Fig. 4); large EKE values are found all along the equator with maximum values in the eastern equatorial Pacific (TIW region). The model EKE generation is related to both barotropic and baroclinic conversions. Fig. 5 presents the time-mean baroclinic and dominant barotropic conversion terms, respectively $w b$ and $-\frac{\partial v}{\partial t} + (-\frac{\partial u}{\partial t})$, not negligible at the surface but has only weak vertical extent). Barotropic conversion is composed of narrow bands of large values, i.e., a primary band near the equator (in the region of shear between EUC and NEC) and a secondary band around 4°N (in the region of shear between SEC and NECC); baroclinic conversion presents a broader pattern between the equator and 6°N. These results are similar to those presented by Baturin and Niiler (1997). The same approximate ratio of 3/2 for barotropic/baroclinic conversion as Masina et al. (1999) is also found here.

Fig. 6 shows the embedded solution when all nested grids are used. It represents a snapshot of sea surface temperature during October 1998. All the known features of the tropical Pacific Ocean are present, i.e., the warm pool in the west and the cold tongue in the east extending from the coast of Peru and producing a westward-propagating tropical instability wave train. TIWs are associated with anticyclonic tropical instability vortices (TIVs; Kennan and Flament, 2000) evidenced on sea surface height in Fig. 7. In the model as in nature, TIW activity is strongest during boreal summer and autumn and are enhanced during La Nina periods, as in 1998. The model TIWs characteristics are consistent with general observations (Willet et al., 2006): TIW wavelength is about 1000 km while TIV size is about 500 km, located between 2°N and 8°N (in situ observations of a vortex is presented in Fig. 9 and commented in the next section), and travelling westward at about 30 km d⁻¹; their wave period at the equator is in the range 15–40 days as in TAO moorings (Fig. 3; see Lyman
Submesoscale structure

In all simulations (FINE, MEDIUM and COARSE), TIWs and TIVs are represented with similar wave scale (or mesoscale) characteristics. The main differences arise at frontal scale (submesoscale). This is clearly illustrated in Fig. 8 showing that a simulated TIW can have the same location and mesoscale structure in all 3 simulations, i.e., the TIW can travel through 2 nested interfaces without alteration of its wave properties. In TIWs, the cusps of the SST front is due to northward advection, on the leading edge of TIVs, of cold water from the equatorial upwelling zone. Peak currents on this edge exceed 1.5 m s⁻¹. TIVs are also associated with a depression of the thermocline of about 50 m.

All fields at all resolutions indicate strong frontogenesis resulting from straining in the vicinity of TIVs. Frontal dynamical features (in particular, asymmetry of vertical velocity and vertical vorticity) appear as predicted by the semigeostrophic theory of frontogenesis (Hoskins, 1982; Thomas and Lee, 2005): downward velocities on the cold side of fronts (maxima near −100 m d⁻¹) are much stronger than upward velocities on the warm side (maxima around 40 m d⁻¹); cyclonic vorticity on the cold side is much stronger than anticyclonic vorticity on the warm side (due to asymmetry of vortex stretching/squashing \( \nabla \times (f + \zeta) \frac{\partial u}{\partial y} \) across the front); vertical heat flux is positive on both sides of the front but much stronger on the cold side. The frontal \( w \) pattern is attributed to the ageostrophic secondary cross-frontal circulation which attempts at restoring geostrophic balance and also results from the divergence of Ekman transport affected by frontal vorticity changes (Stern, 1965; Niiler, 1969).

The kinematics of model TIVs are also in general agreement with in situ observations (Johnson, 1996; Flament et al., 1996; Kennan and Flament, 2000). Fig. 9 presents surface velocity and vorticity observations from the second cruise of the 1990 Tropical Instability Wave Experiment (TIWE-2; Kennan and Flament, 2000). This gridded (at 1/4° resolution) quasi-synoptic observation (sampled over ~20 days and mapped using a translating frame to recover the vortex coherence) is smooth but reveals an anticyclonic vortex of about 500 km with speeds approaching 1 m s⁻¹. As in the model (Fig. 8, second row), relative vorticity is intense (compared to \( f \)) with anticyclonic values in the vortex core (where absolute vorticity approaches zero) and mostly cyclonic values on the edges. Note in particular the filament of high positive vorticity along the leading edge of the vortex, similar to the model solution and consistent with frontogenesis theory.

If frontogenesis is obvious at all model solutions, submesoscale meanders and eddies are clearly present only in the MEDIUM and FINER solutions. In the FINE solution, these submesoscale phenomena are more vigorous and, interestingly, most noticeable away from the major straining area in the northern zone between 4°N and 8°N where they seem capable of breaking up forming filaments at the edge of TIVs. Traces of such submesoscale perturbation in the northeast corner of the vortex are also noticeable in the TIWE data (Fig. 9).
Fig. 4. Mean surface Eddy Kinetic Energy (cm$^2$/s$^2$). Top: 1993–2000 Mean EKE from ROMS COARSE solution (36 km resolution grid in the TIW region nested into a 1° grid). Bottom: drifter observation (R. Lumpkin, personal communication); the drifter data was low passed at 5 days to remove inertial and tidal motion; the data density is relatively high in the TIW region.

Fig. 5. EKE conversion at 10-m depth in the COARSE solution (in units of $10^{-5}$ cm$^2$/s$^3$; dashed lines represent negative values). Top: barotropic conversion from mean to eddy kinetic energy $\frac{\partial u}{\partial y}$. Bottom: baroclinic conversion from available potential energy $\frac{\partial w}{\partial y}$. Eddy flow refers here to all perturbations of the 1993–2000 time mean flow except for temporal scales of less than a day which have been filtered. The other barotropic conversion terms are much smaller that the one presented in the top panel (only the term $\frac{\partial w}{\partial y}$ can locally reach significant values), consistent with Masina et al., 1999.
3.3. Kinetic energy spectrum

Much insight into submesoscale dynamical regimes can be gained by exposing its energy spectral characteristics. Fig. 10 shows shell-integrated two-dimensional KE spectra as a function of the horizontal wavenumber magnitude (Capet et al., 2008c, for details on the method). The MEDIUM domain area is used for this spectral analysis (which corresponds to about $400 \times 200$ grid points); Areal mean is removed and KE fields are made periodic by symmetrization (Denis et al., 2002) to allow the Fourier analysis of our regional non periodic signals; 90 fields at daily intervals are processed to obtain an average spectrum. Since the domain of analysis encompasses the size of a tropical instability wave (in the meridional direction), the largest scales that are retained in our analysis represent the TIW (meso) scale. The spectrum in classical log–log scale (top panel) decays approximately linearly with a slope shallower than $-3$ until it strongly decays near the grid scale. A compensated spectrum for a $k^{-2}$ power law ($KE(k)$ multiplied by $k^2$) is shown in the bottom panel of Fig. 10. There is an obvious peak of energy around 1000 km which corresponds to the TIW wavelength. Then, it is evidently flat in the scale range between 500 and 200 km with a steeper decay at smaller scales. The energy budget underlying this spectrum shape is presented in the next section in ways that will expose the location and transitions between mesoscale, submesoscale and dissipation regimes. The sensitivity of KE budget to resolution will also expose the convergence properties of the model.

3.4. KE injection, transfer and dissipation

Following Capet et al., 2008c, we produce a spectral decomposition with respect to horizontal wavenumber of the KE balance for the primitive equations, i.e., we compute and combine co-spectra between velocity and the various terms that enter the momentum equations. As for the computation of power spectra above, the areal mean is removed and symmetrization is performed on each term before the Fourier transform is calculated. The dominant terms of the KE budget at submesoscale are: the conversion of potential to kinetic energy $I$ which represents a local injection of KE; advection $A$ (the horizontal term tends to dominate over the vertical one); and dissipation $D$ (lateral and vertical subgrid-scale mixing). The contribution of 3D pressure work $P$ is most important at TIW scale where
Fig. 8. SST (top row), surface vorticity (second row), 10-m vertical velocity (third row) and 10-m submesoscale vertical T flux (bottom row) in October 1998 (high TIW season) over the area limited by the finest nested domain. The right, center and left columns present respectively the COARSE (36 km), MEDIUM (12 km) and FINE (4 km) solutions.
with velocity-dependent hyperviscosity (Marchesiello et al., 2009). The model is explicit in the third-order upwind which can be split into a non-dissipative, fourth-order, centered scheme and a biharmonic dissipation term $D_V$. The momentum advection scheme of the model is third-order upwind which can be split into a non-dissipative, fourth-order, centered scheme and a biharmonic dissipation term with velocity-dependent hyperviscosity (Marchesiello et al., 2009).

Therefore, the lateral mixing term $D_H$ is the difference between advection terms $A^{up}$ and $A^H$, computed respectively by the upwind and centered discretization schemes.

The advection terms account for both spectral energy transfer and energy flux through the domain. The lateral flux is cancelled by the periodization method used to compute the spectra and the vertical flux is small. Spectral KE transfer between wavenumbers is thus the essential process accounted for by the advection terms. It is computed as the energy transfer rate in $k$ space, which is the integral in $k$ of the advective term of the KE balance, assuming the flux vanishes at the highest wavenumber $k_{max}$:

$$\Pi(k) = \int_{k_{min}}^{k_{max}} Adk.$$  

Fig. 11 (top panel) presents the KE budget terms $I, A, D_H, D_V, and P$ for the MEDIUM resolution in the high TIW season (note that $D_H$ is multiplied by 10 to allow visualization of the dissipation range in the same plot). As previously noted for mid-latitude solutions, KE injection is present on a wide range of scales and largely counteracted by vertical dissipation (Capet et al., 2008c). In the present case, injection is comparatively more concentrated around the $\lambda = 600 \text{ km}$ wavelength, i.e. about half the Rossby deformation wavelength. This is consistent with baroclinic instability analysis (Scott and Wang, 2005; Smith, 2007; Tulloch et al., in press), which shows that the scale of the fastest growing mode at low latitude is smaller than the deformation scale within a factor of two (possibly as a result of the thermocline sloping up toward the equator). Observations also suggest that the injection scale is about half the Rossby deformation wavelength. This is confirmed in our simulations. A difference with mid-latitude dynamics is that the wind work (included in $D_V$) provides a net KE input at

![TIWE surface vorticity and velocity observations from the second cruise of the 1990 Tropical Instability Wave Experiment (TIWE-2; Kennan and Flament, 2000). The gridded data at a horizontal resolution of 1/4° is based on all available velocity data (ADCP and drifters) sampled over ~20 days and mapped using a translating reference frame to recover the vortex coherence; note that some values are missing in the northeast corner of the plot. This figure is to be compared with Fig. 8 (second row) for the same plot). As previously noted for mid-latitude solutions, KE injection is present on a wide range of scales and largely counteracted by vertical dissipation (Capet et al., 2008c). In the present case, injection is comparatively more concentrated around the $\lambda = 600 \text{ km}$ wavelength, i.e. about half the Rossby deformation wavelength. This is consistent with baroclinic instability analysis (Scott and Wang, 2005; Smith, 2007; Tulloch et al., in press), which shows that the scale of the fastest growing mode at low latitude is smaller than the deformation scale within a factor of two (possibly as a result of the thermocline sloping up toward the equator). Observations also suggest that the injection scale is about half the Rossby deformation wavelength. This is confirmed in our simulations. A difference with mid-latitude dynamics is that the wind work (included in $D_V$) provides a net KE input at
TIW scale as noted, for example, by Baturin and Niiler (1997) and Scott and Xu (2009). However, in our simulations the surface input is largely compensated by 3D pressure work which radiates this energy below the surface (Masina et al., 1999; Luther and Johnson, 1990). Note that, over the 3 month study period, there was a small loss of KE (negative $T$) at TIW scale (not shown), which nearly closes the budget. The residual is largest at TIW scale (on the order of the time tendency) and is due to numerical approximations of the offline KE budget and spectral leakage; this residual becomes very small at submesoscale.

The bottom panel of Fig. 11 presents the KE transfer rate $P(k)$. As for mid-latitude cases, $P(k)$ is negative at mesoscale (upward or inverse) and positive at smaller scale (forward toward the dissipation range). The change of sign of energy flux, i.e., the $P(k)$ zero crossing, occurs at wavelength $\approx 600$ km. Here, more clearly than at mid-latitude, the separation scale between forward and inverse KE flux corresponds to that for the center of KE injection. This result is consistent with altimeter estimations of $P(k)$ zero crossing at various latitude (Scott and Wang, 2005). However, contrarily to mid-latitude examples, barotropic instability plays an important role in tropical regions (see 3.1 and Fig. 5). In terms of spectral budget, barotropic instability should provide a contribution to $P(k)$ between the mean flow and eddy scales (Scott and Wang, 2005), i.e., at scales larger than $\approx 1000$ km. This range is poorly represented in our regional analysis and further investigation will be needed to isolate the precise role of barotropic instabilities. In any case, a large part of the mesoscale energy travels down to the submesoscale range, where the transfer rate $P(k)$ reaches a plateau. This plateau indicates the existence of an inertial regime (constant energy flux through that wavenumber range), although not as a (usual) consequence of null KE injection and dissipation, but rather of compensation between them. This is again in contrast with mid-latitude cases for which no clear KE flux plateau is obtained even at horizontal resolution $\approx 1$ km.

Fig. 10. Top: two-dimensional KE spectrum of MEDIUM solution (12 km) for the horizontal velocity fluctuations at 10 m depth as a function of horizontal wavenumber magnitude (corresponding wavelengths are indicated on top); the straight lines indicate $-2$ (dashed) and $-3$ (dot-dash) spectrum slopes. Bottom: compensated spectrum (multiplied by $k^2$) with a linear ordinate scale. In both panels, the diamond symbol marks the location where the model spectrum first begins to decay relative to a $-2$ slope, and the star symbol marks the start of a more rapid tail off; these spectral locations may be interpreted as the limits of pre- and far-dissipation subranges (see text).
(plateau of $I$) sets in approximately where the KE spectrum decays as $k^{-2}$. In this regime, KE injection is compensated by vertical dissipation so that any change in $I$ is accompanied by a similar change in $DV$ and $A$ remains comparatively smaller than either term. Below 200 km and more strongly below 100 km, this inertial-like cascade is broken as the model lateral dissipation becomes an active player ($I + A - DV - DH$) and prevents any KE accumulation at the grid scale (KE transfer is numerically stopped; $P$ decreases). The succession of dynamical regimes described here are similar to the mid-latitude case, but with a spectral shift towards larger scales.

3.5. Resolution convergence

The FINE domain area is now used to study the resolution sensitivity of the spectral KE budget. This domain corresponds to about $330^2$ grid points ($3^2$ times less at each new resolution for the MEDIUM and COARSE solutions). Because of the smaller size of the domain, the analysis now contains fewer TIW events (3 instead of about 10) and spectral leakage, despite the periodization function, is larger at the TIW scale. Nevertheless, the main spectral characteristics described in the previous section are retained and the spectral convergence demonstrated below is deemed meaningful.

Fig. 12 shows the 10 m KE spectra of all three solutions (KE is larger at 10 m than in a mixed layer average but the spectra are qualitatively very similar). The main result of this comparison is the apparent resolution convergence of the spectral KE distribution. Energy increases with resolution in the submesoscale range but mesoscale energy remains nearly constant. In particular, the MEDIUM and FINE spectra cannot be easily distinguished for $k > 100$ km; the increase of resolution from $dx = 12$ km to 4 km only seems to extend the range over which KE spectra are straight (in log scales) by displacing the dissipative range of the model toward smaller scales. The compensated spectra (bottom panel) provides additional details. The COARSE solution has the least energy at all scales and shows no $k^{-2}$ regime. The $k^{-2}$ regime in MEDIUM is less clear than in Fig. 10, perhaps as a result of poor statistical representation or spectral leakage around 500–400 km. Nevertheless, the same transition to a dissipative regime is seen at around 200 km (where the budget is more robust). In the FINE solution, the flat spectral range is well represented and extends further to about 100 km.

Again, spectral convergence appears more clearly than in mid-latitude submesoscale solutions (even when going from 1.5 to 0.75 km horizontal resolution in the case of Capet et al. (2008c)). The spectral shift is a priori consistent with horizontal length

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5 The energy balance between injection and vertical dissipation can be associated with the need for friction to balance cross-frontal Coriolis acceleration due to the ageostrophic circulation (Garrett and Loder, 1981).
scaling varying as $f^{-1}$ since a ratio of order 10 can be expected between eddy scales at mid-latitude and in the 2–6°N band.

The cycle of energy injection, transfer and dissipation for all solutions is presented in Fig. 13. KE injection (top panel) has a similar spectral profile at all resolutions but is weaker in the COARSE solution, contributing to its weaker KE (Fig. 10). At all resolutions, injection peak is situated near the transition point between forward and inverse KE cascade, i.e., around the 600 km wavelength (middle panel). Note that the scale at which this transition occurs is also stable with resolution as previously observed. However, II does not reach a plateau in the COARSE solution while the FINE and MEDIUM solutions have similar patterns up to the MEDIUM dissipation range. Likewise, the COARSE solution may have a KE injection deficit as a result of a wide lateral dissipation range (bottom panel). Note that vertical dissipation at all resolutions (not shown) mirrors, with opposite sign, the spectral pattern of KE injection in the submesoscale range, as shown for the MEDIUM resolution budget (Fig. 11).

Following the method of Skamarock (2004), we estimate the dissipation range and effective resolution of the model by identifying the high wavenumber portion of the KE spectrum where the curve shows a steep decay (exponential tail off). This method gives 30 km, 60 km and 210 km, respectively for the FINE, MEDIUM and COARSE solutions, i.e., about 7$\delta x$. These values may be thought of as the effective ROMS resolution which has a similar dependency to grid-scale $\delta x$ as found by Skamarock (2004) for the Weather Research and Forecast (WRF) model. This would explain that in the COARSE solution, the model dissipation range prevents the formation of submesoscale instabilities since the model effective resolution is coarser in this case than the dominant scales of MLI. However, inspection of the KE dissipation terms (Fig. 13, bottom panel) suggests that the dissipation range starts at even larger scales than estimated with the “Skamarock” method, i.e., ~450 km, 200 km, and 80 km respectively. Therefore, the 7$\delta x$ empirical law may be indicative of the far-dissipation range where energy decays exponentially (Martinez et al., 1997), but our results support the existence of a pre-dissipation range where dissipation is mild (below its maximum) but not negligible. It would explain why the $k^{-2}$ slopes in MEDIUM and FINE solutions are interrupted at scales well above the dissipation maximum. The COARSE dissipation is not even clearly reduced at the largest scales, which would contribute to the COARSE solution having both weaker energy injection and weaker energy at all represented scales.

An alternative way of understanding how dissipation may work in spectral space is to consider that frontal structures project onto a large range of scales, when Fourier transformed, not just near the frontal scale. Therefore, dissipation acting on a front actually affects the full spectral range involved in the representation of that process.
front, from the mesoscale where the initial frontal deformation occurs down to the actual frontal scale.

3.6. Restratification

The equatorial heat budget has often been investigated for its relevance to ocean climate variability (Menkes et al., 2006) but the effect of small-scale dynamics on this budget has not been evaluated. As shown by Boccaletti et al. (2007), the main impact of submesoscale activity (whether related to mesoscale stirring or frontal instabilities) is restratification of the boundary layer through vertical buoyancy or temperature fluxes $F_t$:

$$F_t = - \frac{\partial wT}{\partial z}. \tag{8}$$
Fig. 14 presents a zonal average of \( F_T \) for the three solutions at different resolutions. Maximum absolute values are about 1 K/month, which is of the order of lateral advection by TIWs (Jochum et al., 2007). The restratification process, with its vertical dipole pattern of warming and cooling, is apparent at all resolutions but increases in strength as resolution increases. The main area of restratification is located between 1\(^\circ\)N and 8\(^\circ\)N and centered around 5\(^\circ\)N. This is also the area where submesoscale eddy kinetic energy (EKE) is found (Fig. 15, top panel). In the high resolution runs, EKE is much larger and explains the similar increase in \( F_T \). Between the MEDIUM and FINE solutions, as in the previous analyses, we find relatively small differences in both EKE and \( F_T \). However, we also notice that the differences between the two solutions tend to increase with latitude. In fact, the large values of EKE found between 6–8\(^\circ\)N in the FINE solution coincide with the presence of submesoscale meanders and eddies that are weaker or absent in the other solutions (Fig. 8).

The length scale corresponding to mixed layer eddies (MLEs) is given by a so-called mixed layer deformation radius (Boccaletti et al., 2007) defined as \( NH/f \), where \( N \) is the Brunt–Vaisala frequency in the mixed layer (small by definition and with little geographic variation), \( H \) the mixed layer depth and \( f \) the Coriolis parameter which is a sinus function of latitude \( (f = 2\Omega \sin \phi, \Omega \) is angular speed of the Earth, \( \phi \) is latitude). The MLE length scale
effects remain necessarily small in our model because a restoring time averaged over the high TIW season and zonally averaged over longitudes 132–44W. 

Submesoscale EKE at 10 m (top), and boundary layer depth HBL (bottom) in the FINE, MEDIUM and COARSE solutions. EKE is computed as \( \frac{1}{2} (u^2 + v^2) \) where \( u' \) and \( v' \) are high pass filtered with the operator described in Section 3.2. All fields are time averaged over the high TIW season and zonally averaged over longitudes 132–120°W.

should thus behave as an inverse function of latitude, although there is as yet no modeling or observational evidence of this dependence. In our simulations, typical wavelength of submesoscale meanders arise at 50–200 km, which is an order of magnitude larger than those generated at mid-latitude (10–20 km) and thus consistent with the \( f^{-1} \) MLE length scaling. Following this law, the size of MLEs should decrease rapidly with latitude away from the equator. At 6°N, it is around 90 km so that only the FINE solution (30–100 km effective resolution) may be able to properly resolve submesoscale turbulence in the 6–8°N latitude band.

Most importantly, Fig. 14 suggests that the presence of finer and better resolved eddy structures keeps modifying the surface heat balance until horizontal resolution has reached about 10 km. As a result of increased submesoscale restratification, the deepening of the boundary layer induced by TIW stretching around 5°N is partially compensated when cross-frontal circulations are at work (Fig. 14, overimposed black line; and Fig. 15, bottom panel). These effects remain necessarily small in our model because a restoring term modifies the applied air-sea flux to maintain SST close to large-scale observations. We may expect larger sensitivities using less constrained, more realistic surface forcing, which would impact on the observed strong coupling between the atmosphere and the ocean surface (Chelton et al., 2001). Note that our reasoning assumes that surface mesoscale activity itself is not overly altered by submesoscale dynamics (through upscaling effects) which could otherwise modify the heat budget in deeper ways. Nevertheless, and even though our submesoscale solutions are for a restricted subregion of the central Pacific, they support the idea that submesoscale upscaling effects are indeed rather weak.

4. Discussion and conclusion

In this paper, the submesoscale-resolving model used in mid-latitude by Capet et al. (2008c) was applied to the near equatorial region of central Pacific where tropical instability waves are the dominant “mesoscale” phenomena. A suite of model solutions with increasing resolution shows that the conventional mid-latitude KE spectrum is in this case strongly shifted towards larger scales with a magnitude consistent with a latitudinal dependence of eddy scales. As a consequence of the general shift of eddy length scales, the resolution convergence of KE spectrum in TIWs is achieved with a grid mesh size of 12 km, and is manifested in both energy injection and spectral energy fluxes at the submesoscale range. It implies that mesoscale dynamics associated with TIWs and their impact on the equatorial upwelling are almost fully captured at about 10 km resolution. However, the restratification process by mixed layer eddies is not fully captured at this resolution, in particular in the northern area of TIWs where these eddies are vigorous. Yet, MLI perturbations with wavelengths as large as 50–200 km instead of 10–20 km at mid-latitude develop at a model resolution of 12 km, instead of 1 km. Here again, the differences agree with a \( f^{-1} \) law of MLE scaling using the mixed layer deformation radius. This result is of importance to applications of MLI parametrization in global models (Fox-Kemper et al., in press).

TIWs are a major contributor to the heat and biogeochemical balance of the equatorial upwelling region (Hansen and Paul, 1984; Gorgues et al., 2005). Mid-latitude submesoscale-resolving simulations tend to show that lateral heat (or buoyancy) flux is dominated by mesoscale eddies, while vertical buoyancy flux is dominated by submesoscale dynamics (Capet et al., 2008b). This general statement appears to be true also for TIWs. It was already known that the effect of instability waves (at the wave scale) is a warming of the equatorial cold tongue through meridional exchanges with tropical waters (Baturin and Niiler, 1997; Menkes et al., 2006); in the present study, the TIW submesoscale effect, as in mid-latitude region, is to restratify the mixed layer (which sees as much as 20% height reduction around 6°N). MLEs appear particularly efficient in this process as suggested by the comparison between MEDIUM and FINE solutions. Addressing more precisely the submesoscale effects on heat and biogeochemical budgets should be considered in future studies.

A relevant aspect of our results for ocean modeling is the spectral decomposition of subgrid-scale dissipation. It is often assumed in oceanography that hyperviscous operators (high-order filters) are useful artifacts to avoid “wasting resolution” by dissipating scales around the mesh size (Frisch et al., 2008, for a critical review). The use of high-order filters are only appropriate in conjunction with high-order discretization methods with low dispersion errors (Durran, 2010). Therefore, in practice, additional filters (spatial or temporal) to hyperviscous operators are often needed. Skamarock (2004) has provided a simple way of estimating the range of scales that can be effectively modified by model dissipation. It concerns not only the viscous operator but more generally...
the ensemble of numerical filters used in the model (Durrant, 1991, for the Robert-Asselin temporal filter example). In regional models with high-order schemes, such as ROMS or WRF, both numerical dispersion and dissipation are reduced to increase the effective resolution at a given mesh size; in this case, according to the method of Skamarock (2004), the model effective resolution is estimated between 5–10 \( \Delta x \). However, our spectral decomposition of KE lateral dissipation (an equivalent biharmonic operator with velocity dependent viscosity) provides new insight which shows that dissipation can be far greater than estimated from the KE spectra alone. When hyperviscosity is used, as here, the far-dissipation range may well be made closer to the grid scale (as \( k^3 \) instead of \( k^5 \)) but the advantage of widening the quasi-inertial range may be partly offset by the maintenance of an intermediate spectral region where dissipation may be mild but not negligible (Frisch et al., 2008). As a result, MEDIUM and FINE solutions have their submesoscale \( k^2 \) slopes interrupted at scales well above the dissipation maximum (in the far-dissipation range), and the COARSE solution presents an energy deficit at all represented scales. Note that in the pre-dissipation range, KE seems to decay as \( k^3 \) (Fig. 10) but this is not confirmed by the compensated spectrum \( k E(k) \) (not shown); a fake impression of QG inertial stratification may thus be conveyed by excessive numerical dissipation.

The embedded approach with 2-way nesting communication, available in the AGRIF version of ROMS, has proven here very useful to address small scale phenomena and analyze their resolution sensitivity at relatively low computational cost (and with little burden of building configurations). A large part of the success is in the techniques developed by Debreu et al. (2011), preserving conservation and continuity at the grids interface. However, there is a downside of using smaller regional domains for spectral decomposition as windowing or periodization methods can reach their limits of validity (Denis et al., 2002). In our case, we noticed a budget residual at large scale (above 300 km) which can be attributed to both offline computation and spectral analysis errors. For this reason, we systematically checked the consistency of our results on various domains and resolutions and ensured at least that our analysis provides a robust, if not exact, interpretation of underlying dynamics. Another caveat is the limited statistical reliability of our solutions at the TIW scale over only 3 months during which the temporal tendency is not negligible. More reliability would be achieved by multiple years of simulation. Nevertheless, we believe that the results will remain qualitatively similar.

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